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EVALUATION AND ASSESSMENT OF ROLL-FORCE AND TORQUE MODELS FOR A HOT PLATE MILL

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EVALUATION AND ASSESSMENT OF ROLL-FORCE AND TORQUE MODELS FOR A HOT PLATE MILL

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ABSTRACT

The purpose of this study is to test the predictive abilities of the theoretical model of Sims (1) and the empirical model of Schultz et al (2) for in-line control of drafting (thickness reduction) in the hot rolling of plate. The prediction of these two models are compared by using the mill operating data for several microalloyed steels rolled on the pilot-scale mill at CANMET. The mean yield stress equations of Gittins et al (3) and Tarokh et al (18), and also the mean strain-rate equations of Sims (1) and Ford & Alexander (5), are compared in combination with Sims' model.

Comparison results show that Schultz's model, in spite of its simpler form, predicts roll force and torque more accurately than Sims' model. The empirical model also can be more easily and rapidly computed and can be updated with new empirical data; thus it is more applicable to in-line adaptive control and optimization of drafting within the operating limits of the rolling process.

LABORATOIRES DE RECHERCHE EN MÉTALLURGIE PHYSIQUE
RAPPORT MRP/PMRL 84-29(TR)

ÉVALUATION DES MODÈLES PRÉVOYANT L'EFFORT ET LE COUPLE UTILISÉS LORS DU LAMINAGE À CHAUD

par

I.C. Ozsoy et G.E. Ruddle

RÉSUMÉ

Cette étude a pour but la vérification des capacités prévisionnelles du modèle théorique de Sims (1) et du modèle empirique de Schultz et al (2) pour le contrôle direct de l'étirage (réduction de l'épaisseur) lors du laminage à chaud des tôles. On compare les prédictions de ces deux modèles à l'aide des données recueillies lors du laminage de plusieurs aciers micro-alliés au laminoir expérimental du CANMET. On compare les équations de Gittins et al (3) et de Tarokh et al (18) relatives à la limite élastique moyenne et également les équations de Sims (1) et de Ford et Alexander (5) relatives à la vitesse de déformation moyenne en utilisant le modèle de Sims.

Les résultats obtenus indiquent que le modèle de Schultz, en dépit de sa forme simplifiée, peut prédire l'effort et le couple de laminage de façon plus précise que le modèle de Sims. De plus, il est possible d'utiliser le modèle empirique plus facilement et plus rapidement et de le mettre à jour en y introduisant de nouvelles données empiriques; par conséquent, il se prête davantage aux ajustements du contrôle direct et à l'optimisation de l'étirage dans les limites opérationnelles du procédé de laminage.

INTRODUCTION

Computer programs that are used to control hot plate mills must be able to predict roll separating forces and roll torques accurately and quickly; therefore, to make use of one of several mathematical models published in the technical literature, evaluation of their predictive abilities is necessary. An objective of this study is to evaluate two commonly used methods for predicting roll forces and torques for application to in-line control of drafting (thickness reduction) in the hot rolling of plate in the pilot-scale rolling mill at the Physical Metallurgy Research Laboratories (PMRL) of CANMET.

The mechanics of flat rolling has been studied extensively (5-11). Some of these models are based on the study of forces acting on the deforming plate during its passage through the roll gap, while others use regression analysis of experimental data to develop empirical relations. Sims' equations (1) on flat rolling are chosen as one of the models because it is used in several industrial computer models for on-line control of plate mills. The empirical relations of Schultz and Smith (2) are included in the present evaluation because of their simplicity.

Constitutive equations, giving functional relationships of stress, strain, strain rate, temperature, chemical composition, and thermomechanical history of the material being rolled form a part of a mathematical model to control the rolling process. Again, a large number of publications can be found in the technical literature describing the interdependence of these parameters (3,4,12-14). In each of these publications, regression analysis is used to develop empirical formulae that define the material's flow stress as a function of variables considered important by the authors. In this study, two empirical equations for mean yield stress proposed by Gittins et al (3) and by Tarokh et al (18) are chosen and combined with Sims' rolling model. In the correlation, the flow stress properties of the steel under investigation are determined in terms of reduction, mean strain-rate and temperature. Two mean strain-rate formulae given by Sims (1) and Ford & Alexander (5) are combined with the mean yield stress equations. The coefficients of the two mean yield stress equations and of Schultz's model are determined by multiple linear regression using the PMRL mill data measured in

hot rolling of Ti-N-Nb, C-Mn, Nb and Va steels. Data from approximately 400 passes are included in the regression analyses.

SIMS MODEL

The roll separating force P and roll torque G required to pass hot steel between the working rolls can be expressed in terms of the resistance of the steel to hot-rolling deformation, respectively, in the general form:

$$P = \bar{k}_p \cdot Y_p \; ; \quad G = \bar{k}_g \cdot Y_g \qquad \text{Eq 1}$$

where $\bar{k}_p$ and $\bar{k}_g$ are mean yield stress functions, and Y_p and Y_g are functions of the roll-pass geometry. A simplified approach due to Sims (1) gives the roll force and torque as:

$$P = \bar{k}_p \sqrt{R'(h_1-h_2)}.Q_p.W$$

$$G = \bar{k}_g.2RR'.Q_g.W \qquad \text{Eq 2}$$

where R, R' are the roll radius and deformed roll radius; h_1, h_2 are the entry and exit thicknesses; Q_p and Q_g are geometric functions of exit thickness, reduction and roll radius; and W is the width of material. The deformed roll radius is determined using the Hitchcock correction for roll flattening (15).

The mean yield stress functions for each type of steel can be expressed as:

$$\bar{k} = f(T,r,\bar{\dot{\varepsilon}}) \qquad \text{Eq 3}$$

where T is temperature, r is reduction ratio $(h_1-h_2)/h_1$, h_1 and h_2 are the entry and exit thicknesses in the reduction, and $\bar{\dot{\varepsilon}}$ is the mean strain rate which is given by Sims (1) as:

$$\bar{\lambda} = \frac{2\pi N}{60} \cdot \sqrt{\frac{R'}{h_2}} \sqrt{\frac{1-r}{r}} \ln\left(\frac{1}{1-r}\right) \qquad \text{Eq 4}$$

and is given by Ford & Alexander (5) as:

$$\bar{\lambda} = \frac{\pi N \sqrt{r}}{30} \sqrt{\frac{r}{h_1}} \left[\frac{4-3r}{(2-r)^2}\right] \qquad \text{Eq 5}$$

where N is the roll speed in rpm. $\bar{k}_p$ and $\bar{k}_g$ are expressed by Gittins et al (3) as:

$$\bar{k}_p \text{ or } \bar{k}_g = A_0 + r^n \left(A_1 + A_2 \ln\bar{\lambda} + \frac{A_3}{T}\right) \qquad \text{Eq 6}$$

and are expressed by Tarokh et al (18) as:

$$\bar{k}_p \text{ or } \bar{k}_g = A_0 + \frac{A_1}{T^2} + A_2 \sqrt{r} + A_3 \frac{\sqrt{r}}{T} + A_4 \sqrt{r} \cdot \ln\bar{\lambda} \qquad \text{Eq 7}$$

where T is temperature (°C/1000), n is a strain-hardening exponent, and the A_i are regression coefficients.

The geometric functions can be expressed as:

$$Q = f(h_2, r, R', Y, \phi) \qquad \text{Eq 8}$$

where y is the workpiece thickness at the neutral point angle ϕ. These functions are defined as:

$$Q_p = \sqrt{\frac{1-r}{r}} \left[0.5\pi \left(\tan^{-1}\sqrt{\frac{r}{1-r}} - 0.5 B\alpha\right) - B\left(\ln\frac{Y}{h_2} - \frac{1}{2}\ln\frac{1}{1-r}\right)\right] \qquad \text{Eq 9}$$

and

$$Q_g = 0.5\alpha - \phi \qquad \text{Eq 10}$$

where

$$\phi = \frac{1}{B} \tan [0.5 \tan^{-1}\sqrt{\frac{r}{1-r}} + \frac{\pi \ell n(1-r)}{8B}] \qquad \text{Eq 11}$$

$$y = h_2[1 + 2B^2(1-\cos\phi)] \qquad \text{Eq 12}$$

$$B = \sqrt{\frac{R'}{h_2}} \qquad \text{Eq 13}$$

$$\alpha = \cos^{-1}\left(1 - \frac{h_1-h_2}{2R'}\right) \qquad \text{Eq 14}$$

This method of calculation offers advantages over previous methods in that (4):

(a) Suitable data on new grades of steel can be readily applied to predict P and G.

(b) Decreases of temperature from pass to pass during the rolling process can now be included in the calculations of P and G.

(c) The calculation can be readily incorporated in mill control programs.

SCHULTZ ET AL MODEL

In a rolling reduction schedule to be calculated for the mill using a theoretical equation, the exit thickness (h_2) is not known and is one of the quantities to be computed (17). A theoretical type equation, as formulated by Sims (1) for example, is capable of determining the roll separating force for a known thickness reduction. It is also necessary, however, to be able to determine the reduction for a given roll separating force when using shape control on the last passes. In this case, the equation requires an iterative process to determine the reduction (or exit thickness) for a specified force because it cannot be solved explicitly.

One of the constraints imposed upon the rolling process control is due to the high speed of the rolling operation and the small time interval

between rolling passes that is available for thickness reduction calculations. On-line corrections to a preprogrammed thickness-reduction schedule must be computed and implemented between passes. This means that the control computer must be able to perform all the mill sequencing operations as well as the schedule calculations in a very short time interval. An iterative process involves several computation steps to obtain the desired result, and consequently may require too much time to be adapted to on-line control.

An empirical roll-force equation that enables explicit computation of pass thickness reduction in an acceptably short time interval was developed by Schultz et al (2). It includes all important process variables, and products and powers of these variables to account for any interaction. The empirical equation which Schultz has simplified to consist of only the most significant terms is expressed as follows:

$$\ell np = Y = B_0 + B_1 \ell n\left(\frac{R}{h_1}\right) + B_2 \ell n\left(\frac{\Delta h}{h_1}\right) + B_3 \ell n\left(\frac{\Delta h}{h_1}\right)\ell n\left(\frac{R}{h_1}\right)$$

$$+ B_4 \frac{T}{1000} + B_5 \frac{T}{1000} \ell n\left(\frac{\Delta h}{h_1}\right) + B_6 \ell n\left(\frac{\Delta h}{h_1}\right)\ell n\left(\frac{R}{h_1}\right)^2 \qquad \text{Eq 15}$$

and

$$P = [\text{antilog } Y] \sqrt{R\Delta h}.W \qquad \text{Eq 16}$$

$$\Delta h = h_1 - h_2 \qquad \text{Eq 17}$$

p = antilog Y = roll pressure

where B_0 through B_6 are constant coefficients which can be evaluated by multiple regression analysis of available rolling data. All other parameters are as previously defined in Sims' model.

The torque expression in Schultz's model is also given as follows:

$$G = (C_0 + C_1 h_1)\, P \sqrt{R\Delta h} \qquad \text{Eq 18}$$

where the quantity $(C_0 + C_1 h_1)$ is an effective arm factor and $\sqrt{R\Delta h}$ is the length of contact between work roll and plate. C_0 and C_1 are constant coefficients which are determined by regression analysis of rolling data. The friction effect is assumed to be negligible on a large mill and is not included in the Schultz model.

EXPERIMENTAL RESULTS

The large pilot-scale rolling mill at PMRL is a two-high, single stand, reversing configuration with the following specifications:

motor drive	225 kW (300 hp)
motor speeds	150/300/450 rpm
peripheral roll speeds	0.5/1.0/1.5 m/s (100/200/300 ft/min)
max. roll separating force	4.5 MN (10^6 lb)
roll diameter, width	470, 457 mm (18.5, 18.0 in.)
roll gap setting	0-130 mm (0-5.0 in.)

Data collection for roll forces, mill motor current, voltage and speed, and slab thicknesses and temperatures is automatically recorded during rolling by a PDP 11/23 microcomputer; this system is explained in the paper of Ruddle et al (16). Data obtained from rolling of plain C-Mn and Ti-N-Nb, Nb and V microalloyed steels, at 45 rpm roll speed and reduction schedules typical of industrial practice for hot-rolled plate, were used to compare the predictive abilities of the theoretical and empirical models. The compositions of the steels are given in Table 1. Mean values $\bar{k}_p$ and $\bar{k}_g$ were computed respectively from the roll force and torque measurements using the transposes of Sims' equations (Eq 2); the data and computed values are presented in Table 2. Measured values for roll torque per unit width of plate were determined by the equation

$$G = \frac{30}{\pi N} \quad \frac{0.8\ VI}{W} \qquad \text{Eq 19}$$

where N is roll speed (rpm), V is the mill motor voltage (volts), I is the mill motor current (amperes), 0.8 VI is the estimated roll power (watts), and W is the plate width (cm).

Tables 3-6 present the coefficients of the mean yield stress equations (Eq 6 and 7), computed by multiple linear regression of the data, as shown, for example, in Table 2, from 100 rolling passes for each of the four steels. The strain-hardening exponent value n=0.02 was used in Eq 6. The coefficients were solved using mean strain rate values determined from the Ford & Alexander equation (Eq 5) and from the Sims' equation (Eq 4).

In Table 7, the Gittins and Tarokh equations for mean yield stress are compared using both the Ford & Alexander and the Sims equations for mean strain rate. The comparison is made by the sums of the squares of the differences of roll forces, and roll torques, determined by Sims' theoretical equations and by measurements in rolling tests. This analysis shows that use of either of the two mean strain rate equations yields approximately the same results, however the Tarokh equation is clearly the better of the two mean yield stress equations as indicated by the smaller values in the table. Table 8 presents a sample of the differences, pass by pass, between the actual (measured) and calculated (Sims) values of roll force and torque for the four steels. An indication of the accuracy of the Sims model is evaluated for each pass by computing

$$\text{per cent difference} = \frac{\text{actual value} - \text{calculated value}}{\text{actual value}} \qquad \text{Eq 20}$$

The coefficients of the Schultz et al equations (Eq 15, 16 and 18) were determined by multiple linear regression of the rolling data from Table 2 for each of the four steels. The values of the coefficients are given in Table 9. The accuracy of the Schultz model was computed as the per cent difference (Eq 20) between the measured values and the Schultz calculations of roll force and torque for each pass in the rolling tests of the four steels. The differences are presented in Table 10.

Finally, the results in Tables 8 and 10 enable a pass-by-pass comparison of both the Sims and Schultz et al predictions with the measured values of roll force and torque. Figures 1-4 present the comparisons of roll force graphically for the four steels. A few general comments can be deduced

from the roll force comparisons. Firstly, the Sims model overestimates the roll force, whereas the estimate of the Schultz et al model is much closer to the actual roll force. Secondly, both models show response to significant changes in rolling strain and temperature conditions. And thirdly, the models can respond to recrystallization and transformation - softening of the steel only to the extent permitted by the regression data, and therefore the estimates do not precisely follow some of the large pass-to-pass changes in measured roll force, as shown for example in passes 16-24, Fig. 4.

The accuracies of the Sims and Schultz et al models are shown for both roll force and torque predictions in Table 11 by sums of the squares of the differences between predicted and measured values. This analysis also shows the Schultz et al predictions for roll force to be considerably more accurate than the Sims. However, the Schultz et al predictions for roll torque are generally only slightly more accurate than Sims. A plot of the roll torque estimates for the four grades of steels in Fig. 5-8 shows the magnitude of difference with the measured values.

The Sims prediction of roll force is less accurate because it is an iterative solution. Usually the deformed roll radius R' is not known, and use of the Hitchcock equation (15) to determine R' requires simultaneous solution with the roll force equation (Eq 2). The iterative solution results in truncation error and also requires increased computation time and memory space. Another reason for the greater difference generally observed between the Sims' predictions and the measured values of roll force and torque is that only the yield stress factor in the Sims equations is empirically formulated by regression of rolling data; the geometric factor is developed from theory. The Schultz et al equation, on the other hand, is entirely formulated by regression of rolling data, and is therefore expected to conform more closely to the measured data. Thus the explicit equation of Schultz et al (Eq 15, 16 and 18) as well as giving more accurate predictions can be more readily and rapidly computed.

The relatively higher computation speed and accuracy of the Schultz et al model renders it applicable to in-line adaptive control of the rolling process (2). The model can make use of feedback data from the mill to update itself as necessary to control roll force and torque and exit thickness at desired values. Moreover, the model can be programmed to optimize the rolling process within the operating limits of the mill.

A computer program written in Fortran IV for the roll force and torque calculations of Sims and of Schultz et al is given in the Appendix.

SUMMARY

The theoretical model given by Sims and the empirical model given by Schultz et al were tested in order to compare their predictive abilities by using the CANMET pilot-scale mill operating data from flat-rolling of four steels. Two different mean yield stress equations proposed by Gittins et al and Tarokh et al were combined with the Sims model. Sims' and Ford & Alexander's mean strain rate expressions were used in the Gittins et al and the Tarokh et al mean yield stress equations. It was seen that by comparison with the measured rolling data, the more accurate results were obtained by using the Tarokh mean yield stress equation in Sims' model. The choice of the mean strain rate equations has no significant effect on the result of the Sims prediction.

The Schultz et al empirical model predicts roll force with considerably greater accuracy than the Sims tneoretical model, however the Schultz prediction of roll torque is only slightly more accurate. The comparative case with which the empirical model can be computed and updated with new data makes it readily applicable to adaptive on-line control and optimization of rolling reduction schedules.

Further work aims to build a mathematical model that includes roll speeds in Eq 15, and thus it will be possible to control roll speed as well as exit thickness.

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Table 1 - Steel compositions (wt %)

Grade	C	Mn	S	P	Si	Al	Mo	Nb	V	Ti	N	Cr
Ti-N-Nb	0.07	1.8	0.003	0.003	0.23	0.03	0.31	0-0.06		0.010	0.006	
C-Mn	0.07	0.6	0.010	0.005	0.20	0.04						
Nb	0.10	1.5	0.003	0.004	0.30	0.025		0.05	0-0.10			0-0.30
V	0.06	1.9					0.08		0.40			

Table 2 - The sample data for Ti-N-Nb, C-Mn, Nb and V steels (roll speed = 45 rpm, motor voltage = 220 V)

Ti-N-Nb steel

Entry thickness, cm	Exit thickness, cm	Temperature, °C	Reduction, cm	Deformed roll radius, cm	Actual roll force, MN/cm	Actual motor current, Amp	Actual roll torque, N-m/cm	Calculated mean yield stress, $\bar{\sigma}_p$ MPa	Calculated mean yield stress, $\bar{\sigma}_s$ MPa
13.653	12.776	1200.000	0.064	23.762	0.0432	985	2315.8	95.3	109.2
12.776	12.154	1097.000	0.049	23.930	0.0503	750	1830.3	132.4	133.5
12.154	11.537	1072.000	0.051	24.026	0.0610	1076	2615.1	160.3	191.2
11.537	10.909	1047.000	0.054	24.045	0.0642	1125	2722.9	166.3	197.3
10.909	10.287	1032.000	0.057	24.108	0.0711	1170	2820.2	183.7	205.0
10.287	9.647	1017.000	0.062	24.073	0.0690	1117	2681.5	174.5	166.6
9.647	9.014	995.000	0.066	24.088	0.0699	1256	3002.9	176.7	187.7
9.014	8.390	977.000	0.069	24.145	0.0758	1230	2928.8	191.2	183.9
8.390	7.765	957.000	0.074	24.182	0.0801	1305	3094.9	200.1	194.2
7.765	7.127	945.000	0.082	24.157	0.0787	1305	3082.4	192.4	191.6
7.127	6.495	930.000	0.089	24.177	0.0804	1350	3176.0	195.1	198.1
6.495	5.867	890.000	0.097	24.206	0.0833	1308	3064.9	199.7	191.9
5.867	5.232	880.000	0.108	24.203	0.0839	1380	3220.8	196.3	200.4
5.232	4.605	870.000	0.120	24.238	0.0871	1395	3242.9	200.6	202.9
4.605	3.998	860.000	0.132	24.395	0.1022	1496	3464.0	232.7	219.6
3.998	3.510	850.000	0.122	24.725	0.1123	1440	3321.3	280.8	269.3
3.510	3.002	837.000	0.145	24.671	0.1118	1638	3763.2	264.2	250.4
3.002	2.487	830.000	0.172	24.617	0.1083	1496	3423.6	242.5	234.5
2.487	1.986	807.000	0.201	24.678	0.1108	1462	3332.8	237.1	231.4
1.986	1.506	780.000	0.242	24.894	0.1258	1447	3285.9	251.8	204.7

Table 2 (Cont'd)

C-Mn Steel

Entry thickness, cm	Exit thickness, cm	Temperature, °C	Reduction, cm	Deformed roll radius, cm	Actual roll force, MN/cm	Actual motor current, Amp	Actual roll torque, N-m/cm	Calculated mean yield stress, $\bar{k}_p$ MPa	Calculated mean yield stress, $\bar{k}_g$ MPa
13.018	12.751	1142.000	0.021	23.991	0.0246	345	845.5	101.7	136.0
12.751	12.507	1122.000	0.019	24.128	0.0288	217	529.9	124.1	88.9
12.507	11.775	1105.000	0.059	23.817	0.0436	858	2088.1	105.2	122.0
11.775	11.265	1092.000	0.043	23.941	0.0423	656	1591.0	122.9	128.1
11.265	10.765	1075.000	0.044	23.979	0.0450	720	1740.3	131.8	141.4
10.765	10.257	1060.000	0.047	23.976	0.0455	708	1705.4	131.4	137.6
10.257	9.754	1047.000	0.049	24.009	0.0481	768	1843.7	139.0	149.4
9.754	9.246	1030.000	0.052	24.003	0.0480	738	1765.6	137.3	142.4
9.246	8.740	1017.000	0.055	24.013	0.0488	772	1840.7	139.1	148.8
8.740	8.235	1000.000	0.058	24.024	0.0498	746	1772.8	141.2	143.2
8.235	7.727	980.000	0.062	24.032	0.0508	825	1953.9	142.6	157.4
7.727	7.231	962.000	0.064	24.090	0.0549	843	1989.9	154.9	162.2
7.231	6.728	942.000	0.070	24.109	0.0576	941	2213.8	159.6	179.0
6.728	6.447	920.000	0.042	24.333	0.0441	480	1125.5	166.6	174.8
6.447	5.852	852.000	0.092	24.064	0.0630	1080	2524.0	156.3	162.8
5.852	5.237	845.000	0.105	24.116	0.0712	1181	2750.9	170.1	174.3
5.237	4.719	842.000	0.099	24.189	0.0671	1117	2718.6	173.5	216.2
4.719	4.214	835.000	0.107	24.210	0.0674	1046	2420.5	173.2	194.8
4.214	3.708	830.000	0.120	24.222	0.0686	1301	3000.7	171.9	241.4
3.708	3.228	820.000	0.129	24.395	0.0808	1113	2558.7	202.0	210.5
3.228	2.713	812.000	0.160	24.291	0.0767	1147	2628.3	177.1	181.2
2.713	2.332	802.000	0.140	24.580	0.0774	1016	2320.6	205.0	213.5
2.332	1.905	792.000	0.183	24.201	0.0562	716	1605.1	132.9	121.9
1.905	1.567	780.000	0.177	24.687	0.0753	757	1717.9	191.8	167.5

Table 2 (Cont'd)

Nb Steel

Entry thickness, cm	Exit thickness, cm	Temperature, °C	Reduction, cm	Deformed roll radius, cm	Actual roll force, MN/cm	Actual motor current, Amp	Actual roll torque, N-m/cm	Calculated mean yield stress, $\bar{k}_p$ MPa	Calculated mean yield stress, $\bar{k}_g$ MPa
12.700	11.788	1150.000	0.072	23.845	0.0592	945	2711.2	126.7	125.1
11.788	10.541	1127.000	0.106	23.802	0.0708	1575	4485.9	126.7	158.4
10.541	9.522	1100.000	0.097	23.863	0.0696	1278	3613.7	137.1	141.1
9.522	8.514	1077.000	0.106	23.884	0.0728	1297	3641.2	142.3	142.8
8.514	7.511	1057.000	0.118	23.926	0.0804	1406	3919.1	154.8	154.0
7.511	6.507	1027.000	0.134	23.956	0.0860	1473	4076.9	161.9	160.1
6.507	5.758	905.000	0.115	24.159	0.0929	1342	3688.3	201.2	186.6
5.758	5.004	895.000	0.131	24.179	0.0963	1477	4031.0	203.0	203.1
5.004	4.364	882.000	0.128	24.269	0.0925	1297	3515.3	208.5	217.6
4.364	3.780	867.000	0.134	24.580	0.1186	1472	3964.9	272.0	255.3
3.780	3.259	855.000	0.138	24.643	0.1119	1533	4098.3	265.6	279.2
3.259	2.746	840.000	0.157	24.618	0.1078	1218	3234.1	248.3	222.0
2.746	2.344	830.000	0.146	24.784	0.0969	975	2571.5	247.8	229.6
2.344	1.976	820.000	0.157	24.980	0.1025	926	2425.9	262.6	225.2
1.976	1.605	805.000	0.188	25.034	0.1069	990	2576.4	256.0	206.4
1.605	1.351	792.000	0.458	25.459	0.0935	686	1773.5	266.5	232.2

Table 2 (Cont'd)

V Steel

Entry thickness, cm	Exit thickness, cm	Temperature, °C	Reduction, cm	Deformed roll radius, cm	Actual roll force, MN/cm	Actual motor current, Amp	Actual roll torque, N-m/cm	Calculated mean yield stress, $\bar{K}_p$ MPa	Calculated mean yield stress, $\bar{K}_g$ MPa
13.335	12.761	1130.000	0.043	23.850	0.0378	682	1671.3	104.4	127.2
12.761	12.502	1120.000	0.020	24.219	0.0350	326	796.8	146.0	129.4
12.502	11.768	1100.000	0.059	23.878	0.0521	982	2394.0	125.2	139.5
11.768	11.257	1090.000	0.043	24.017	0.0496	787	1913.7	144.0	153.9
11.257	10.752	1077.000	0.045	24.041	0.0514	858	2080.9	149.4	168.1
10.752	10.249	1060.000	0.047	24.067	0.0536	776	1877.2	155.3	151.9
10.249	9.743	1047.000	0.049	24.079	0.0550	933	2251.2	158.2	181.7
9.743	9.238	1030.000	0.052	24.102	0.0572	877	2110.7	163.8	170.3
9.238	8.735	1015.000	0.054	24.138	0.0603	967	2321.4	171.9	187.6
8.735	8.232	1002.000	0.058	24.159	0.0623	937	2243.6	176.4	181.2
8.232	7.724	982.000	0.062	24.188	0.0619	1005	2400.3	173.3	192.9
7.724	7.224	962.000	0.065	24.216	0.0673	1031	2456.2	188.2	198.7
7.224	6.718	942.000	0.070	24.221	0.0685	1087	2583.1	188.7	207.9
6.718	6.452	922.000	0.040	24.699	0.0601	611	1448.3	232.0	229.6
6.452	5.842	902.000	0.095	24.157	0.0753	1308	3092.6	183.6	196.6
5.842	5.217	887.000	0.107	24.193	0.0814	1353	3191.0	192.2	200.2
5.217	4.707	872.000	0.098	24.323	0.0789	1188	2794.9	205.0	223.3
4.707	4.199	855.000	0.108	24.338	0.0800	1053	2471.1	204.2	197.9
4.199	3.708	842.000	0.117	24.465	0.0889	1331	3115.7	225.7	253.3
3.708	3.223	827.000	0.131	24.622	0.1023	1350	3152.3	252.7	256.8
3.223	2.723	810.000	0.155	24.638	0.1070	1537	3580.1	249.6	248.8
2.723	2.339	795.000	0.141	24.951	0.1046	1177	2734.3	273.6	249.0
2.339	1.920	785.000	0.179	24.553	0.0830	855	1981.7	196.9	150.8
1.920	1.577	762.000	0.179	25.125	0.1047	1053	2434.7	261.5	233.6

Table 3 - The coefficients of the Gittins et al mean yield stress equation with Ford & Alexander mean strain rate

$$\bar{k}_p \text{ or } \bar{k}_g = A_0 + r^{0.02}(A_1 + A_2 \ln\bar{\lambda} + A_3/T)$$

$$\bar{\lambda} = \frac{\pi N\sqrt{r}}{30}\sqrt{\frac{R}{h_1}}\left[\frac{4-3r}{(2-r)^2}\right]$$

$\bar{k}_p$	A_0	A_1	A_2	A_3
Ti-N-Nb	3465.68	-3977.19	29.78	491.28
C-Mn	-28.79	-157.95	-37.88	364.29
Nb	-725.79	501.81	-19.11	454.00
V	1115.25	-1500.63	-33.87	526.25
$\bar{k}_g$	A_0	A_1	A_2	A_3
Ti-N-Nb	11404.99	-12367.80	169.44	427.95
C-Mn	-3071.01	2948.26	-150.78	559.84
Nb	-2660.38	2555.35	-76.64	488.12
V	-2319.36	2048.64	-160.43	706.10

Table 4 - The coefficients of the Gittins et al mean yield stress equation with Sims' mean strain rate

$$\bar{k}_p \text{ or } \bar{k}_g = A_0 + r^{0.02}(A_1 + A_2 \ln\bar{\lambda} + A_3/T)$$

$$\bar{\lambda} = \frac{\pi N}{30}\sqrt{\frac{R'}{h_2}}\sqrt{\frac{1-r}{r}}\ln\left(\frac{1}{1-r}\right)$$

$\bar{k}_p$	A_0	A_1	A_2	A_3
Ti-N-Nb	3410.70	-3921.40	26.91	494.59
C-Mn	121.72	-311.16	-31.68	355.08
Nb	-646.31	420.20	-16.46	450.49
V	1266.66	-1653.76	-27.54	515.86
$\bar{k}_g$	A_0	A_1	A_2	A_3
Ti-N-Nb	11027.75	-11986.78	150.56	453.03
C-Mn	-2798.90	2666.94	-138.44	549.17
Nb	-2558.86	2451.39	-71.04	483.32
V	-2021.03	1741.21	-146.67	693.47

Table 5 - The coefficients of the Tarokh et al mean yield stress equation with Ford & Alexander mean strain rate

$$\bar{k}_p \text{ or } \bar{k}_g = A_0 + \frac{A_1}{T^2} + A_2 \frac{\sqrt{\bar{r}}}{T} + A_3 \frac{\sqrt{\bar{r}}}{T} + A_4 \sqrt{\bar{r}}\, \ell n \bar{\lambda}$$

$$\bar{\lambda} = \frac{\pi N \sqrt{r}}{30} \sqrt{\frac{R}{h_1}} \left[\frac{4-3r}{(2-r)^2}\right]$$

$\bar{k}_p$	A_0	A_1	A_2	A_3	A_4
Ti-N-Nb	-180.11	765.63	2679.87	-4787.22	842.58
C-Mn	-85.86	294.34	918.63	-1239.87	97.59
Nb	-212.01	326.87	895.58	-659.50	-73.02
V	-110.30	404.78	1147.21	-1755.92	193.97
$\bar{k}_g$	A_0	A_1	A_2	A_3	A_4
Ti-N-Nb	3.26	966.32	2408.69	-6266.74	1362.59
C-Mn	-168.20	332.11	1141.72	-1113.34	-92.74
Nb	-255.49	321.64	1107.68	-578.57	-195.10
V	-182.70	417.37	1425.33	-1587.64	-27.86

Table 6 - The coefficients of the Tarokh et al mean yield stress equation with Sims' mean strain rate

$$\bar{k}_p \text{ or } \bar{k}_g = A_0 + \frac{A_1}{\bar{T}^2} + A_2\sqrt{\frac{\bar{r}}{\bar{T}}} + A_3\frac{\bar{r}}{\bar{T}} + A_4\sqrt{\bar{r}}\,\ell n\bar{\lambda}$$

$$\bar{\lambda} = \frac{\pi N}{30}\sqrt{\frac{R'}{h_2}}\sqrt{\frac{1-r}{r}}\,\ell n\left(\frac{1}{1-r}\right)$$

$\bar{k}_p$	A_0	A_1	A_2	A_3	A_4
Ti-N-Nb	-220.99	824.91	3044.63	-5258.84	859.61
C-Mn	-95.54	311.65	1049.24	-1420.34	122.16
Nb	-214.38	333.58	927.43	-712.42	-62.76
V	-122.85	425.02	1325.66	-1990.03	222.11
$\bar{k}_g$	A_0	A_1	A_2	A_3	A_4
Ti-N-Nb	-21.46	974.53	2432.79	-6196.20	1259.47
C-Mn	-173.21	344.03	1229.19	-1243.78	-66.63
Nb	-254.95	325.67	1124.30	-617.83	-179.13
V	-189.19	431.06	1544.46	-1755.06	1.78

Table 7 - Comparison of Gittins et al and the Tarokh et al mean yield stress equations combined with Sims' and Ford & Alexander mean strain rates for four grades of steel. P_i(MN/cm) and G_i(N-m/cm) are the Sims calculated roll force and torque values per unit width, $\bar{P}_i$ and $\bar{G}_i$ are the measured roll force and torque values per unit width. n is the number of rolling passes.

Yield stress model	Gittins et al mean yield stress model				Tarokh et al mean yield stress model			
Mean strain rate	Sims'		Ford & Alexander		Sims'		Ford & Alexander	
Sum of squares of differences	$\sum_{i=1}^{n}(P_i-\bar{P}_i)^2$	$\sum_{i=1}^{n}(G_i-\bar{G}_i)^2$	$\sum_{i=1}^{n}(P_i-\bar{P}_i)^2$	$\sum_{i=1}^{n}(G_i-\bar{G}_i)^2$	$\sum_{i=1}^{n}(P_i-\bar{P}_i)^2$	$\sum_{i=1}^{n}(G_i-\bar{G}_i)^2$	$\sum_{i=1}^{n}(P_i-\bar{P}_i)^2$	$\sum_{i=1}^{n}(G_i-\bar{G}_i)^2$
Ti-N-Nb n=100	0.012186	0.139764×10^8	0.012171	0.138415×10^8	0.009088	0.097306×10^8	0.009292	0.096211×10^8
C-Mn n=95	0.008464	0.088259×10^8	0.008428	0.087921×10^8	0.007601	0.076439×10^8	0.007639	0.076471×10^8
Nb n=96	0.012652	0.115220×10^8	0.012641	0.114874×10^8	0.012529	0.111306×10^8	0.012530	0.111050×10^8
V n=96	0.010008	0.130596×10^8	0.010003	0.131965×10^8	0.008335	0.099491×10^8	0.008398	0.100218×10^8

Table 8 - Comparison of roll force and torque solutions for Sims' model* with measured roll forces and torques

Ti-N-Nb Steel

Roll force, MN/cm		Difference,	Difference,	Torque, N-m/cm		Difference,	Difference,
actual	Calculated	MN/cm	%	actual	Calculated	N-m/cm	%
0.043	0.056	-0.012	-28.740	2315.800	2431.974	-116.174	-5.017
0.050	0.059	-0.008	-16.894	1830.300	2648.293	-817.993	-44.692
0.061	0.063	-0.002	-3.201	2615.100	2776.629	-161.529	-6.177
0.064	0.067	-0.003	-4.755	2722.900	2912.883	-189.983	-6.977
0.071	0.070	0.001	1.069	2820.200	2985.157	-164.957	-5.849
0.069	0.073	-0.004	-6.425	2681.500	3062.296	-380.796	-14.201
0.070	0.077	-0.008	-10.809	3002.900	3166.358	-163.458	-5.443
0.076	0.081	-0.005	-7.060	2928.800	3248.816	-320.016	-10.927
0.080	0.085	-0.005	-6.384	3094.900	3340.638	-245.738	-7.940
0.079	0.088	-0.009	-11.905	3082.400	3373.790	-291.390	-9.453
0.080	0.092	-0.011	-13.897	3176.000	3421.457	-245.457	-7.728
0.083	0.098	-0.015	-17.485	3064.900	3575.907	-511.007	-16.673
0.084	0.100	-0.016	-19.627	3220.800	3550.686	-329.885	-10.242
0.087	0.104	-0.017	-19.178	3242.900	3545.839	-302.939	-9.342
0.102	0.109	-0.007	-6.527	3464.000	3585.472	-121.472	-3.507
0.112	0.115	-0.003	-2.835	3321.300	3655.496	-334.196	-10.062
0.112	0.121	-0.009	-7.840	3763.200	3705.966	57.234	1.521
0.108	0.128	-0.020	-18.505	3423.600	3805.762	-382.162	-11.163
0.111	0.138	-0.027	-24.459	3332.800	3918.379	-585.579	-17.570
0.126	0.151	-0.025	-20.247	3285.900	4097.443	-811.543	-24.698

Table 8 (Cont'd)

C-Mn Steel

Roll force, MN/cm		Difference,	Difference,	Torque, N-m/cm		Difference,	Difference,
actual	Calculated	MN/cm	%	actual	Calculated	N-m/cm	%
0.025	0.028	-0.004	-15.038	845.500	670.483	175.017	20.700
0.029	0.029	0.000	0.813	529.900	646.620	-116.720	-22.027
0.044	0.050	-0.006	-14.835	2088.100	2152.337	-64.237	-3.076
0.042	0.044	-0.001	-2.870	1591.000	1525.350	65.650	4.126
0.045	0.045	0.000	0.393	1740.300	1558.970	181.330	10.419
0.045	0.047	-0.001	-2.758	1705.400	1643.115	62.285	3.652
0.048	0.048	0.000	0.044	1843.700	1679.221	164.479	8.921
0.048	0.050	-0.002	-4.634	1765.600	1766.684	-1.084	-0.061
0.049	0.052	-0.003	-6.031	1840.700	1814.118	26.582	1.444
0.050	0.054	-0.004	-8.074	1772.800	1884.533	-111.733	-6.303
0.051	0.056	-0.006	-11.125	1953.900	1985.216	-31.316	-1.603
0.055	0.058	-0.004	-6.433	1989.900	2024.381	-34.481	-1.733
0.058	0.062	-0.004	-6.924	2213.800	2145.878	67.922	3.068
0.044	0.052	-0.008	-17.358	1125.500	1334.023	-208.523	-18.527
0.063	0.079	-0.016	-26.020	2524.000	3106.933	-582.933	-23.096
0.071	0.082	-0.011	-14.889	2750.900	3194.541	-443.641	-16.127
0.067	0.079	-0.012	-17.210	2718.600	2764.105	-45.505	-1.674
0.067	0.080	-0.012	-18.249	2420.500	2712.456	-291.956	-12.062
0.069	0.081	-0.013	-18.471	3000.700	2689.971	310.729	10.355
0.081	0.083	-0.002	-2.586	2558.700	2588.656	-29.956	-1.171
0.077	0.087	-0.010	-13.345	2628.300	2658.969	-30.669	-1.167
0.077	0.083	-0.006	-7.483	2320.600	2157.464	163.136	7.030
0.056	0.089	-0.033	-57.845	1605.100	2249.920	-644.820	-40.173
0.075	0.089	-0.013	-17.680	1717.900	1919.677	-201.776	-11.746

Table 8 (Cont'd)

Nb Steel

Roll force, MN/cm		Difference,	Difference,	Torque, N-m/cm		Difference,	Difference,
actual	Calculated	MN/cm	%	actual	Calculated	N-m/cm	%
0.059	0.053	0.006	9.757	2711.200	2588.548	122.652	4.524
0.071	0.074	-0.003	-4.620	4485.900	4212.108	273.792	6.103
0.070	0.070	-0.000	-0.368	3613.700	3501.168	112.532	3.114
0.073	0.075	-0.003	-3.492	3641.200	3697.286	-56.086	-1.540
0.080	0.081	-0.001	-0.997	3919.100	3899.141	19.959	0.509
0.086	0.090	-0.004	-4.722	4076.900	4233.435	-156.535	-3.840
0.093	0.106	-0.013	-13.812	3688.300	4188.849	-500.549	-13.571
0.096	0.112	-0.015	-16.017	4031.000	4361.807	-330.807	-8.207
0.092	0.108	-0.016	-16.994	3515.300	3814.177	-298.877	-8.502
0.119	0.111	0.008	6.541	3964.900	3634.022	330.878	8.345
0.112	0.111	0.001	1.030	4098.300	3356.770	741.530	18.094
0.108	0.118	-0.011	-9.913	3234.100	3470.571	-236.471	-7.312
0.097	0.111	-0.014	-14.086	2571.500	2803.323	-231.823	-9.015
0.102	0.113	-0.011	-10.318	2425.900	2659.070	-233.170	-9.612
0.107	0.125	-0.018	-16.795	2576.400	2823.116	-246.716	-9.576
0.094	0.111	-0.018	-18.957	1773.500	2045.657	-272.157	-15.346

Table 8 (Cont'd)

V Steel

Roll force, MN/cm		Difference,	Difference,	Torque, N-m/cm		Difference,	Difference,
actual	Calculated	MN/cm	%	actual	Calculated	N-m/cm	%
0.038	0.050	-0.012	-31.120	1671.300	1940.446	-269.146	-16.104
0.035	0.037	-0.002	-4.842	796.800	883.171	-86.371	-10.840
0.052	0.059	-0.007	-13.392	2394.000	2658.132	-264.132	-11.033
0.050	0.052	-0.003	-5.715	1913.700	1895.802	17.898	0.935
0.051	0.054	-0.003	-5.053	2080.900	1932.296	148.604	7.141
0.054	0.056	-0.003	-4.944	1877.200	2002.310	-125.110	-6.665
0.055	0.058	-0.003	-6.023	2251.200	2075.460	175.740	7.807
0.057	0.061	-0.004	-6.260	2110.700	2152.337	-41.637	-1.973
0.060	0.063	-0.003	-4.622	2321.400	2218.970	102.430	4.412
0.062	0.065	-0.003	-4.753	2243.600	2283.286	-39.686	-1.769
0.062	0.069	-0.007	-11.023	2400.300	2408.386	-8.086	-0.337
0.067	0.072	-0.005	-6.827	2456.200	2481.494	-25.294	-1.030
0.068	0.076	-0.007	-10.669	2583.100	2618.559	-35.459	-1.373
0.060	0.064	-0.004	-7.208	1448.300	1557.156	-108.855	-7.516
0.075	0.088	-0.013	-17.011	3092.600	3324.260	-231.660	-7.491
0.081	0.092	-0.011	-13.384	3191.000	3461.020	-270.020	-8.462
0.079	0.091	-0.012	-15.307	2794.900	3024.950	-230.051	-8.231
0.080	0.095	-0.015	-18.801	2471.100	3099.171	-628.071	-25.417
0.089	0.098	-0.009	-10.433	3115.700	3065.149	50.551	1.622
0.102	0.103	-0.000	-0.371	3152.300	3084.676	67.624	2.145
0.107	0.109	-0.002	-1.684	3580.100	3174.930	405.170	11.317
0.105	0.108	-0.003	-3.129	2734.800	2703.271	31.529	1.153
0.083	0.114	-0.031	-37.093	1981.700	2791.283	-809.583	-40.853
0.105	0.119	-0.014	-13.497	2434.700	2532.898	-98.198	-4.033

*Using mean values of k_p, k_g determined by the Tarokh et al equations and mean values of λ determined by the Sims' equation.

Table 9 - The coefficients of the Schultz et al model

$$Y = B_0 + B_1 \ln\left(\frac{R}{h_1}\right) + B_2 \ln\left(\frac{\Delta h}{h_1}\right) + B_3 \ln\left(\frac{\Delta h}{h_1}\right)\ln\left(\frac{R}{h_1}\right) + B_4 \frac{T}{1000} + B_5 \frac{T}{1000} \ln\left(\frac{\Delta h}{h_1}\right) + B_6 \ln\left(\frac{\Delta h}{h_1}\right)\ln\left(\frac{R}{h_1}\right)^2$$

$$G = (C_0 + C_1 h_1)\, P\sqrt{R\Delta h}$$

	B_0	B_1	B_2	B_3	B_4	B_5	B_6	C_0	C_1
Ti-N-Nb	-10.61	1.35	-2.80	-201.06	3.67	1.99	100.62	8780.14	210.51
C-Mn	1.44	-1.18	2.98	65.48	-4.66	-2.20	-33.16	9141.28	182.65
Nb	3.18	-0.81	2.71	-386.35	-5.91	-1.95	192.91	8413.30	239.38
V	-3.33	-0.10	0.47	269.07	-1.15	-0.34	-134.70	8424.96	260.50

Table 10 - Comparison of roll force and torque solutions for the Schultz et al model with measured roll forces and torques

Ti-N-Nb Steel

Roll force, MN/cm		Difference,	Difference,	Torque, N-m/cm		Difference,	Difference,
actual	Calculated	MN/cm	%	actual	Calculated	N-m/cm	%
0.043	0.046	-0.003	-5.747	2315.800	2416.968	-101.168	-4.369
0.050	0.056	-0.006	-12.125	1830.300	2473.134	-642.834	-35.122
0.061	0.061	0.000	0.688	2615.100	2615.607	-0.507	-0.019
0.064	0.065	-0.000	-0.837	2722.900	2787.570	-64.670	-2.375
0.071	0.068	0.003	4.767	2820.200	2867.443	-47.243	-1.675
0.069	0.070	-0.001	-1.989	2681.500	2987.231	-305.731	-11.401
0.070	0.074	-0.004	-6.325	3002.900	3098.924	-96.024	-3.198
0.076	0.078	-0.002	-2.646	2928.800	3181.364	-252.563	-8.623
0.080	0.081	-0.001	-1.463	3094.900	3284.847	-189.947	-6.137
0.079	0.083	-0.005	-5.954	3082.400	3362.662	-280.262	-9.092
0.080	0.086	-0.006	-7.494	3176.000	3424.075	-248.075	-7.811
0.083	0.092	-0.008	-9.965	3064.900	3570.825	-505.925	-16.507
0.084	0.093	-0.010	-11.290	3220.800	3612.425	-391.625	-12.159
0.087	0.096	-0.009	-10.282	3242.900	3643.452	-400.552	-12.352
0.102	0.100	0.002	2.498	3464.000	3669.239	-205.239	-5.925
0.112	0.110	0.003	2.371	3321.300	3572.368	-251.068	-7.559
0.112	0.112	0.000	0.226	3763.200	3668.756	94.444	2.510
0.108	0.115	-0.007	-6.276	3423.600	3768.647	-345.047	-10.078
0.111	0.121	-0.010	-9.084	3332.800	3858.401	-525.601	-15.771
0.126	0.128	-0.002	-1.677	3285.900	3951.503	-665.603	-20.256

Table 10 (Cont'd)

C-Mn Steel

Roll force, MN/cm		Difference,	Difference,	Torque, N-m/cm		Difference,	Difference,
actual	Calculated	MN/cm	%	actual	Calculated	N-m/cm	%
0.025	0.030	-0.005	-22.137	845.500	866.937	-21.437	-2.535
0.029	0.028	0.001	3.618	529.900	762.409	-232.509	-43.878
0.044	0.047	-0.004	-8.480	2088.100	2241.339	-153.239	-7.339
0.042	0.042	0.001	1.316	1591.000	1631.830	-40.830	-2.566
0.045	0.042	0.003	5.733	1740.300	1628.408	111.892	6.429
0.045	0.044	0.001	3.036	1705.400	1693.191	12.209	0.716
0.048	0.046	0.002	5.083	1843.700	1728.933	114.767	6.225
0.048	0.047	0.001	1.264	1765.600	1788.617	-23.017	-1.304
0.049	0.049	-0.001	-1.220	1840.700	1844.705	-4.005	-0.218
0.050	0.051	-0.001	-2.958	1772.800	1896.613	-123.813	-6.984
0.051	0.053	-0.002	-4.919	1953.900	1960.395	-6.495	-0.332
0.055	0.055	-0.000	-0.282	1989.900	1983.495	6.405	0.322
0.058	0.058	-0.000	-0.166	2213.800	2075.272	138.528	6.257
0.044	0.048	-0.004	-8.357	1125.500	1273.410	-147.910	-13.142
0.063	0.064	-0.001	-1.140	2524.000	2458.586	65.414	2.592
0.071	0.069	0.002	2.538	2750.900	2693.514	57.385	2.086
0.067	0.070	-0.002	-3.584	2718.600	2448.730	269.870	9.927
0.067	0.073	-0.005	-8.136	2420.500	2511.597	-91.097	-3.764
0.069	0.076	-0.008	-11.116	3000.700	2605.106	395.594	13.183
0.081	0.078	0.003	3.547	2558.700	2569.963	-11.263	-0.440
0.077	0.079	-0.002	-2.711	2628.300	2666.865	-38.565	-1.467
0.077	0.079	-0.001	-1.442	2320.600	2264.065	56.535	2.436
0.056	0.074	-0.018	-31.989	1605.100	2248.062	-642.962	-40.057
0.075	0.072	0.003	4.216	1717.900	1928.902	-211.002	-12.283

Table 10 (Cont'd)

Nb Steel

Roll force, MN/cm		Difference,	Difference,	Torque, N-m/cm		Difference,	Difference,
actual	Calculated	MN/cm	%	actual	Calculated	N-m/cm	%
0.059	0.052	0.007	12.414	2711.200	2749.318	-38.118	-1.406
0.071	0.068	0.003	3.704	4485.900	4146.536	339.364	7.565
0.070	0.066	0.004	5.151	3613.700	3533.016	80.684	2.233
0.073	0.071	0.001	1.819	3641.200	3719.697	-78.497	-2.156
0.080	0.077	0.003	3.646	3919.100	3930.835	-11.735	-0.299
0.086	0.086	0.000	0.003	4076.900	4265.461	-188.561	-4.625
0.093	0.094	-0.001	-1.220	3688.300	3933.635	-245.335	-6.652
0.096	0.103	-0.006	-6.672	4031.000	4234.031	-203.031	-5.037
0.092	0.100	-0.008	-8.212	3515.300	3730.924	-215.624	-6.134
0.119	0.103	0.016	13.506	3964.900	3594.238	370.662	9.349
0.112	0.103	0.009	8.031	4098.300	3355.472	742.827	18.125
0.108	0.109	-0.002	-1.483	3234.100	3492.059	-257.959	-7.976
0.097	0.102	-0.005	-5.249	2571.500	2843.335	-271.835	-10.571
0.102	0.102	0.000	0.415	2425.900	2693.893	-267.993	-11.047
0.107	0.106	0.001	0.949	2576.400	2778.287	-201.887	-7.836
0.094	0.096	-0.002	-2.336	1773.500	2056.607	-283.107	-15.963

Table 10 (Cont'd)

V Steel

Roll force, MN/cm		Difference,	Difference,	Torque, N-m/cm		Difference,	Difference,
actual	Calculated	MN/cm	%	actual	Calculated	N-m/cm	%
0.038	0.047	-0.009	-23.853	1671.300	2045.935	-374.635	-22.416
0.035	0.036	-0.001	-2.423	796.800	1039.102	-242.302	-30.409
0.052	0.054	-0.002	-4.581	2394.000	2643.507	-249.507	-10.422
0.050	0.050	-0.000	-0.722	1913.700	1989.259	-75.559	-3.948
0.051	0.052	-0.000	-0.484	2080.900	2020.781	60.119	2.889
0.054	0.054	-0.000	-0.019	1877.200	2068.336	-191.136	-10.182
0.055	0.056	-0.001	-1.458	2251.200	2134.909	116.291	5.166
0.057	0.058	-0.001	-1.453	2110.700	2191.653	-80.953	-3.835
0.060	0.060	-0.000	-0.062	2321.400	2246.942	74.458	3.207
0.062	0.063	-0.001	-0.837	2243.600	2311.141	-67.541	-3.010
0.062	0.066	-0.004	-6.145	2400.300	2399.428	-0.872	0.036
0.067	0.068	-0.001	-1.486	2456.200	2443.542	12.658	0.515
0.068	0.072	-0.003	-4.421	2583.100	2542.229	40.871	1.582
0.060	0.064	-0.004	-7.071	1448.300	1637.036	-188.736	-13.032
0.075	0.080	-0.005	-6.212	3092.600	3060.097	32.503	1.051
0.081	0.084	-0.003	-3.232	3191.000	3203.302	-12.302	-0.386
0.079	0.085	-0.006	-7.478	2794.900	2872.311	-77.411	-2.770
0.080	0.088	-0.008	-10.187	2471.100	2939.458	-468.358	-18.953
0.089	0.091	-0.002	-2.261	3115.700	2939.480	176.220	5.656
0.102	0.094	0.009	8.507	3152.300	2967.403	184.897	5.865
0.107	0.096	0.011	10.416	3580.100	3044.106	535.994	14.971
0.105	0.097	0.007	6.891	2734.800	2672.384	62.416	2.282
0.083	0.097	-0.014	-16.801	1981.700	2748.265	-766.565	-38.682
0.105	0.098	0.007	6.485	2434.700	2480.993	-46.293	-1.901

Table 11 - Comparison of the Sims* and Schultz et al models for four grade of steels. P_i (MN/cm) and G_i (N-m/cm) are the calculated roll force and torque values per unit width, $\bar{P}_i$ and $\bar{G}_i$ are the measured roll force and torque values per unit width

Grade	Sims'		Schultz	
	$\sum_{i=1}^{n} (P_i-\bar{P}_i)^2$	$\sum_{i=1}^{n} (G_i-\bar{G}_i)^2$	$\sum_{i=1}^{n} (P_i-\bar{P}_i)^2$	$\sum_{i=1}^{n} (G_i-\bar{G}_i)^2$
Ti-N-Nb n=100	0.009088	0.097306×10^8	0.002338	0.108648×10^8
C-Mn n=96	0.007601	0.076439×10^8	0.004648	0.065745×10^8
Nb n=96	0.012529	0.111306×10^8	0.004472	0.101363×10^8
V n=96	0.008335	0.099491×10^8	0.004433	0.089775×10^8

*Using mean values of k_p, k_g determined by the Tarokh et al equations and mean values of λ determined by the Sims' equation.

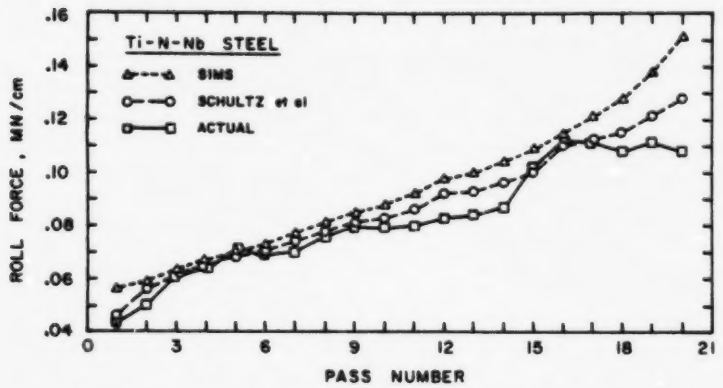

Figure 1. Roll separating force for Ti-N-Nb steel. □ : Measured , 0 : Schultz et al., Δ : Sims. (neg 12568)

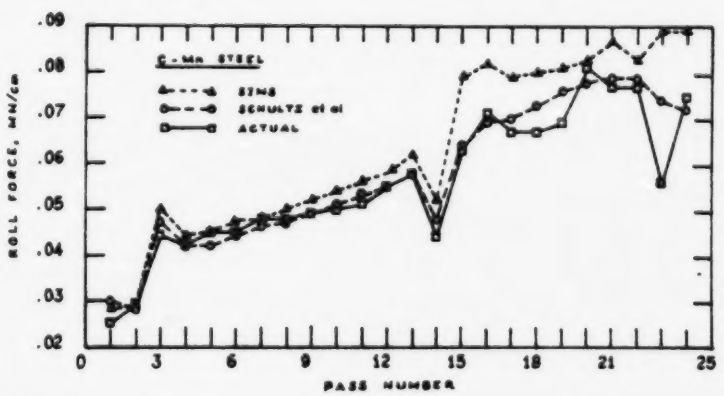

Figure 2. Roll separating force for C-Mn steel. □ : Measured , 0 : Schultz et al., Δ : Sims. (neg 12569)

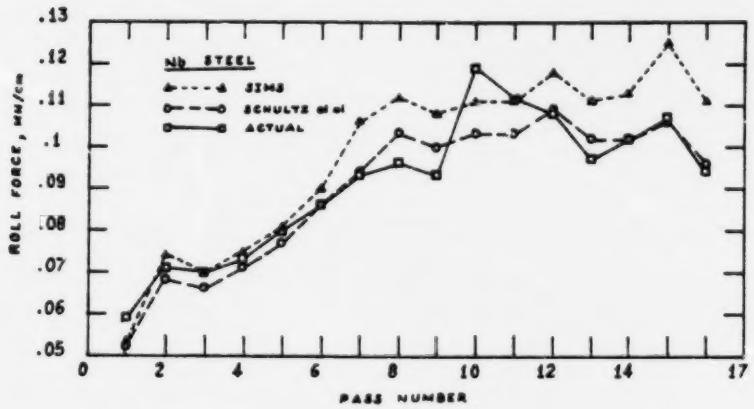

Figure 3. Roll separating force for Nb steel. □ : Measured , 0 : Schultz et al., Δ : Sims. (neg 12574)

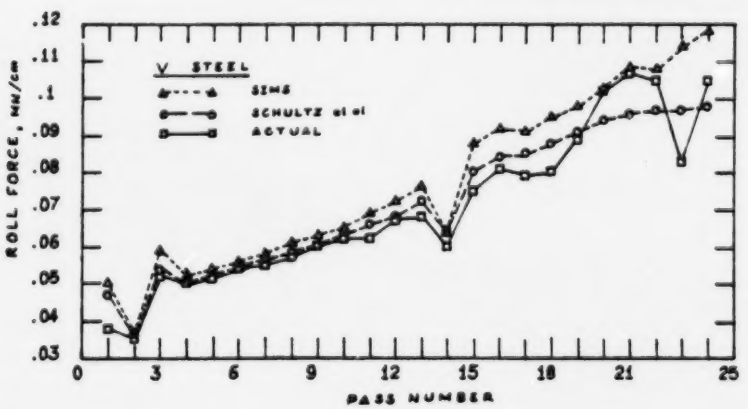

Figure 4. Roll separating force for V steel. □ : Measured , 0 : Schultz et al., Δ : Sims. (neg 12571)

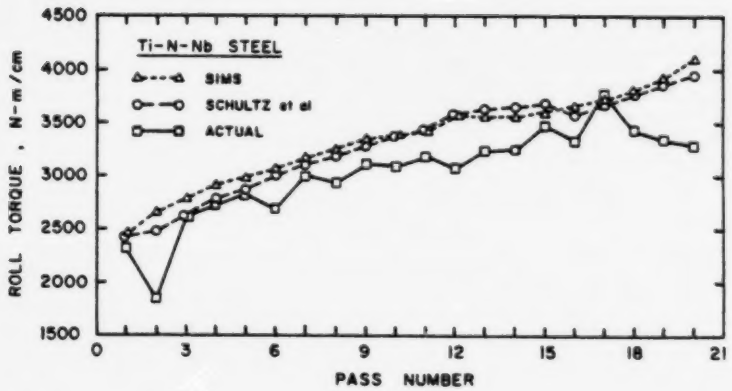

Figure 5. Roll torque for Ti-N-Nb steel. □ : Measured , 0 : Schultz et al., Δ : Sims. (neg 12575)

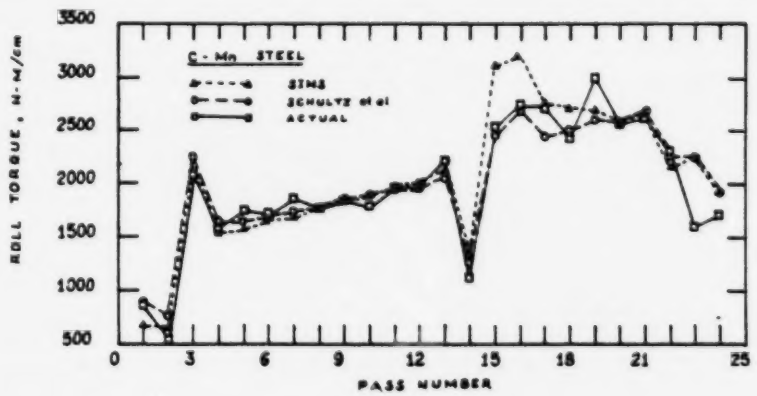

Figure 6. Roll torque for C-Mn steel. □ : Measured, 0 : Schultz et al., Δ : Sims. (neg 12573)

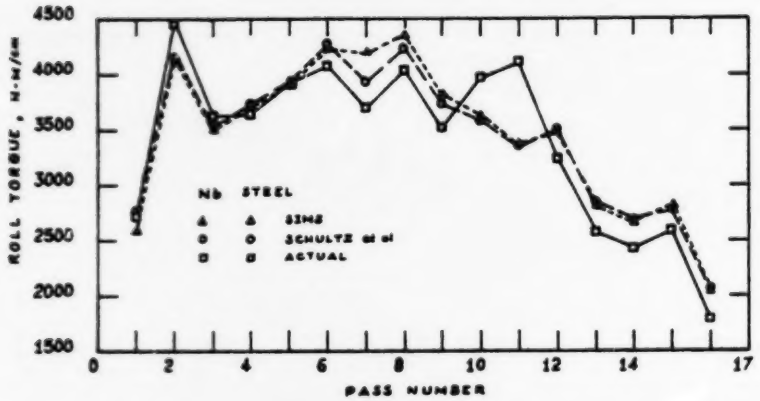

Figure 7. Roll torque for Nb Steel. □ : Measured , 0 : Schultz et al., Δ : Sims. (neg 12572).

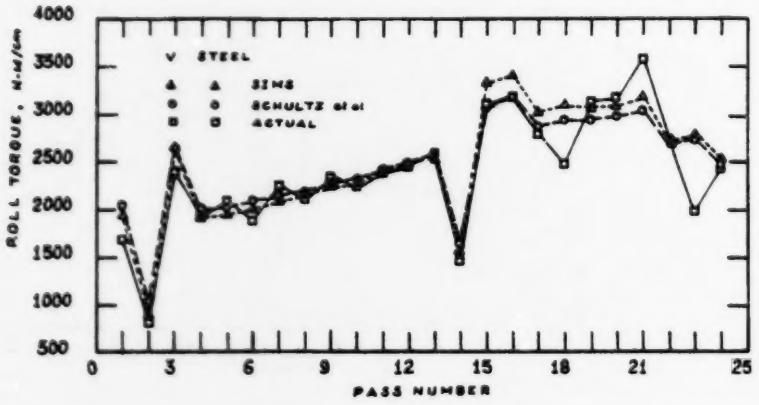

Figure 8. Roll torque for V steel. □ : Measured , 0 : Schultz et al., Δ : Sims. (neg 12570).

APPENDIX

```
      DIMENSION  TH1(100),TH2(100),TEMP(100)
      DIMENSION  RAD(100),RED(100),TORQ(100),YCSH(100),YTSH(100)
      DIMENSION  BB(25),FSMIT(100),TSMIT(100),P(100),DELT(100)
      DIMENSION  PP(100),GG(100),QQP(100),QQG(100),TANGN(100)
      DIMENSION  AFI(100),ALF(100),Y1(100,10),Y2(100,10),F(100)
      DIMENSION  E1(100),E2(100),DIFF(100),DIFT(100),CKP(5),CKG(5)
      DIMENSION  E3(100),E4(100),DIFF1(100),DIFT1(100)
C
      CALL ASSIGN(1,'DX1:FTN2 .DAT',-1)
      CALL ASSIGN(3,'DX1:FTN2 .DAT',-1)
      CALL ASSIGN(4,'DX1:FTN2 .DAT',-1)
      CALL ASSIGN(5,'DX1:FTN2 .DAT',-1)
      CALL ASSIGN(2,'DX1:FTN2 .OUT',-1)
C
C
      READ(1,709) IR1,IR2,IR3
  709 FORMAT(3I5)
C
      IF(IR1.EQ.1) WRITE(2,710)
      IF(IR1.EQ.2) WRITE(2,702)
      IF(IR1.EQ.3) WRITE(2,703)
      IF(IR1.EQ.4) WRITE(2,704)
  710 FORMAT(//,'TI-N STEEL')
  702 FORMAT(//,'C-MN STEEL')
  703 FORMAT(//,'NB STEEL')
  704 FORMAT(//,'V STEEL')
C
      IF(IR2.EQ.1) WRITE(2,705)
      IF(IR2.EQ.2) WRITE(2,706)
  705 FORMAT(//,'GITTINS YIELD STRESS MODEL')
  706 FORMAT(//,'BUCKMAN YIELD STRESS MODEL')
C
      IF(IR3.EQ.1) WRITE(2,707)
      IF(IR3.EQ.2) WRITE(2,708)
  707 FORMAT(//,'SIMS MEAN STRAIN RATE')
  708 FORMAT(//,'FORD & ALEXANDER MEAN STRAIN RATE')
C
      READ(1,201) N,N1,N2,M1,M2,M3,M4,M5,M6,RADIS
  201 FORMAT(9I5,F6.2)
C
      READ(1,300) (BB(I),I=1,10)
  300 FORMAT(4D15.6)
C
```

```
      DO 233 I=1,N
      READ(1,234) (Y1(I,J),J=1,N2)
  234 FORMAT(8D9.4)
  233 CONTINUE
C
      DO 239 I=1,N
      READ(5,238) (Y2(I,J),J=1,N2)
  238 FORMAT(8D9.4)
  239 CONTINUE
C
      READ(3,235) (YCSH(I),I=1,N)
  235 FORMAT(F6.1)
C
      READ(4,236) (YTSH(I),I=1,N)
  236 FORMAT(F6.1)
C
C
      DO 237 I=1,N
      TH1(I)=Y1(I,M1)
      TH2(I)=Y1(I,M2)
      F(I)=Y1(I,M3)
      RAD(I)=Y1(I,M4)
      TEMP(I)=Y1(I,M5)
      TORQ(I)=Y2(I,M6)
  237 CONTINUE
C
C
      WRITE(2,261)
  261 FORMAT(//,'CALCULATED COEFFICIENTS OF SCHULTZ MODEL')
      WRITE(2,262) (BB(I),I=1,9)
  262 FORMAT(//,4F12.2)
C
      WRITE(2,263)
  263 FORMAT(//,'CALCULATED COEFFICIENTS OF KP')
      WRITE(2,264) (CKP(I),I=1,5)
  264 FORMAT(//,4F12.2)
      WRITE(2,265)
  265 FORMAT(//,'CALCULATED COEFFICIENTS OF KG')
      WRITE(2,266) (CKG(I),I=1,5)
  266 FORMAT(//,4F12.2)
C
C     CALCULATION OF ROLLING PARAMETERS WITH SIMS
C
      DO 400 I=1,N
C
      RED(I)=(TH1(I)-TH2(I))/TH1(I)
      ASQR1=SQRT(TH2(I)/(TH1(I)-TH2(I)))
      ASQR2=SQRT(RAD(I)/TH2(I))
      AOG3=ALOG(1./(1.-(TH1(I)-TH2(I))/TH1(I)))
      ASQR3=SQRT((TH1(I)-TH2(I))/TH2(I))
      ASQR4=SQRT(RAD(I)*(TH1(I)-TH2(I)))
      ATAG1=ATAN(ASQR3)
```

```
C
      CCX=(1.-((TH1(I)-TH2(I))/(2.*RAD(I))))
      SSY=SQRT(1.-CCX*CCX)
      ALF(I)=ATAN(SSY/CCX)
C
      TANGN(I)=((3.14/8.)*((TH2(I)/RAD(I))**0.5)*
     1 ALOG(TH2(I)/TH1(I))+(0.5*ATAG1))
      AFI(I)=SIN(TANGN(I))/COS(TANGN(I))*SQRT(TH2(1)/RAD(I))
C
      AOG2=TH2(I)*(1.+(RAD(I)/TH2(I))*(1.-COS(AFI(I))))
C
      QQP(I)=((3.14/2.)*(ASQR1*ATAG1)-(3.14/4.*ALF(I)*ASQR1*
     1 ASQR2)-(ASQR1*ASQR2*ALOG(AOG2/TH2(I)))+(0.5*ASQR1*
     2 ASQR2*AOG3))
C
      PP(I)=1.15*1.0E-04*YCSH(I)*ASQR4*QQP(I)
C
      QQG(I)=(ALF(I)/2.)-AFI(I)
C
      GG(I)=2.*(RADIS*RAD(I))*(1.15*YISH(I))*QQG(I)
  400 CONTINUE
C
      STFN=0.
      STRQN=0.
      DO 411 I=1,N
      STFN=STFN+((F(I)-PP(I))*(F(I)-PP(I)))
      STRQN=STRQN+((TORQ(I)-GG(I))*(TORQ(I)-GG(I)))
  411 CONTINUE
C
C
C     CALCULATION OF ROLLING PARAMETERS WITH SCHULTZ
C
      DO 301 I=1,N
C
      DELT(I)=TH1(I)-TH2(I)
C
      P(I)=BB(1)+(BB(2)*ALOG(RADIS/TH1(I)))+(BB(3)*
     1 ALOG(DELT(I)/TH1(I)))+(BB(4)*ALOG(DELT(I)/TH1(I))*
     2 ALOG(RADIS/TH1(I)))+(BB(5)*TEMP(I)/1000.)+(BB(6)*
     3 (TEMP(I)/1000.)*ALOG(DELT(I)/TH1(I)))+(BB(7)*
     4 ALOG(DELT(I)/TH1(I))*ALOG((RADIS/TH1(I))**2.))
C
      P(I)=EXP(P(I))
C
      FSMIT(I)=P(I)*SQRT(RADIS*DELT(I))
C
      TSMIT(I)=(BB(8)+BB(9)*TH1(I)+BB(10)*TH2(I))*
     1 SQRT(RADIS*DELT(I))*FSMIT(I)
C
  301 CONTINUE
C
      TFSMI=0.
      TTSMI=0.
      DO 306 I=1,N
      TFSMI=TFSMI+((F(I)-FSMIT(I))*(F(I)-FSMIT(I)))
      TTSMI=TTSMI+((TORQ(I)-TSMIT(I))*(TORQ(I)-TSMIT(I)))
  306 CONTINUE
```

```
C
      WRITE(2,420)
  420 FORMAT(//,'DIFFERENCE BETWEEN OBSERVATIONS AND CALCULATED VALUES'
C
      WRITE(2,422)
  422 FORMAT(//,'SIMS MODEL WITHOUT INTEGRATION')
      WRITE(2,406) STFN
  406 FORMAT(//,'SUM OF SQUARES OF ERRORS FOR FORCE =',F12.8)
      WRITE(2,407) STRQN
  407 FORMAT(//,'SUM OF SQUARES OF ERRORS FOR TORQUE =',F15.3)
C
      WRITE(2,423)
  423 FORMAT(//,'SMITH MODEL')
      WRITE(2,307) TFSMI
  307 FORMAT(//,'SUM OF SQUARES OF ERRORS FOR FORCE =',F12.8)
      WRITE(2,408) TTSMI
  408 FORMAT(//,'SUM OF SQUARES OF ERRORS FOR TORQUE =',F15.3)
C
      DO 701 I=1,N
      DIFF(I)=F(I)-PP(I)
      DIFF1(I)=F(I)-FSMIT(I)
      E1(I)=((F(I)-PP(I))/F(I))*100.
      E3(I)=((F(I)-FSMIT(I))/F(I))*100.
      DIFT(I)=TORQ(I)-GG(I)
      DIFT1(I)=TORQ(I)-TSMIT(I)
      E2(I)=((TORQ(I)-GG(I))/TORQ(I))*100.
      E4(I)=((TORQ(I)-TSMIT(I))/TORQ(I))*100.
  701 CONTINUE
C
      WRITE(2,604)
  604 FORMAT(///,4X,'TH1(I)',5X,'TH2(I)',1X,'TEMP(I)',
     1 6X,'RED(I)',3X,'RAD(I)',3X,'KPC(I)',5X,'KGC(I)')
C
      DO 600 I=1,N
      WRITE(2,601) Y1(I,1),Y1(I,2),Y1(I,3),RED(I)
     1 ,RAD(I),YCSH(I),YTSH(I)
  601 FORMAT(/,7F10.3)
  600 CONTINUE
C
      WRITE(2,682)
  682 FORMAT(///,'SIMS MODEL')
C
      WRITE(2,605)
  605 FORMAT(///,3X,'F(I)',4X,'FH(I)',2X,'DIFF(I)',2X,'E1(I)',
     1 4X,'TORQ(I)',3X,'TORQH(I)',2X,'DIFT(I)',2X,'E2(I)')
C
      DO 602 I=1,N
      WRITE(2,603) F(I),PP(I),DIFF(I),E1(I),TORQ(I),GG(I),
     1 DIFT(I),E2(I)
  603 FORMAT(/,3F8.3,4F10.3,F8.3)
  602 CONTINUE
C
      WRITE(2,683)
  683 FORMAT(///,'SMITH MODEL')
C
      WRITE(2,680)
  680 FORMAT(///,3X,'F(I)',4X,'FH(I)',2X,'DIFF(I)',2X,'E3(I)',
     1 4X,'TORQ(I)',3X,'TORQH(I)',2X,'DIFT(I)',2X,'E4(I)')
```

```
C
      DO 684 I=1,N
      WRITE(2,681) F(I),FSMIT(I),DIFF1(I),E3(I),TORQ(I),
     1 TSMIT(I),DIFT1(I),E4(I)
  681 FORMAT(/,3F8.3,4F10.3,F8.3)
  684 CONTINUE
C
C
      STOP
      END
```